theory for heat flux was obtained within 2 per cent. Figure 1 shows that within this accuracy no specific value for the condensation coefficient can be concluded. However, values as low as $f = 0{\cdot}10$ can be excluded, and for the present range of vapor pressures the condensation coefficient is indicated to lie in the range $0{\cdot}25 < f < 1{\cdot}00$. A narrower range could have been determined if operation at lower vapor pressures had succeeded.

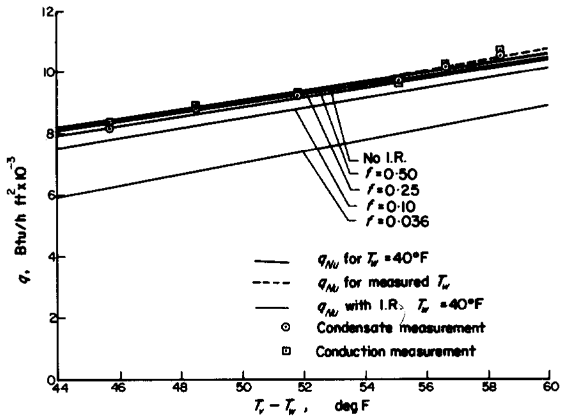

FIG. 1. Heat flux vs. temperature difference (I.R. = interfacial resistance).

It has been found in the course of the experimentations that noncondensable gases are continuously generated. It is therefore felt that some of the low condensation coefficients quoted in the literature for this class of liquids may be attributed to the presence of noncondensable gas.

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A NOTE ON NATURAL CONVECTION AT HIGH PRANDTL NUMBERS

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NOMENCLATURE

x, y, distances along and perpendicular to the plate;
u, v, velocity components along x and y directions;
Gr_x, local Grashof number;
g, acceleration due to gravity;
t, temperature;
f, non-dimensional stream function;
F, transformed stream function in the inner layer;
G, transformed stream function in the outer layer;
c_p, specific heat at constant pressure.

Greek symbols
γ, a constant;
ψ, stream function;
σ, Prandtl number; ν/α;
ε, local dissipation number defined by (2);
β, co-efficient of volume expansion of the fluid;
η, similarity variable;
ρ, density;
μ, viscosity;
ν, kinematic viscosity;
α, thermal diffusivity;
θ, temperature excess;
ϕ, non-dimensional temperature function;

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ζ, stretched similarity variable;
Φ, transformed temperature function.

Subscripts
w, wall condition;
∞, condition at large distance from the plate;
0, no dissipation;
1, first-order dissipation effects.

1. INTRODUCTION

GEBHART [1] investigated effects of viscous dissipation in natural convection about semi-infinite flat vertical surfaces (isothermal). He started with the usual equations for conservation of mass, momentum and energy, namely,

$$\left.\begin{aligned}
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\\
&\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} \pm g\rho\beta\theta,\\
&u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \alpha\frac{\partial^2 \theta}{\partial y^2} + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2,\\
&u = v = 0,\quad \theta = t_\omega - t_\infty,\quad y = 0,\\
&u \to 0,\quad \theta \to 0 \quad \text{as} \quad y \to \infty,
\end{aligned}\right\} \quad (1)$$

where u, v are the velocity components, θ is the temperature excess $(t - t_\infty)$, x is measured from the leading edge along the plate and y is the distance out perpendicular to the plate, the plus and minus signs apply for heating and for cooling of the fluid respectively and the other symbols have their usual meanings. He then developed perturbation-type similar solutions of (1) given by

$$\left.\begin{aligned}
&u = \frac{\partial\psi}{\partial y},\quad v = -\frac{\partial\psi}{\partial x},\\
&\phi = \frac{t - t_\infty}{t_\omega - t_\infty} = \phi_0 \pm 4\epsilon\phi_1 \pm \ldots,\\
&\psi = 2\sqrt{(2)}\,v(Gr_x)^{\frac{1}{4}}(f_0 \pm 4\epsilon f_1 \pm \ldots),\\
&Gr_x = |g\beta x^3(t_\omega - t_\infty)/v^2|,\\
&\eta = \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{\frac{1}{4}},\\
&\epsilon = \frac{g\beta x}{c_p},
\end{aligned}\right\} \quad (2)$$

where f's and ϕ's are functions of η alone satisfying ordinary differential equations discussed in detail in [1]. Assuming $f_1 = 0$, Gebhart solved these equations for $\sigma = 10^{-2}$, 0·72, 10^2 and 10^4 and observed that the ratio $\phi_1'(0)/\phi_0'(0)$ closely approached an asymptotic value in the Prandtl-number interval 10^2 to 10^4. We shall in what follows obtain solutions for large values of σ in powers of $\sigma^{-\frac{1}{2}}$ and substantiate Gebhart's conjecture.

2. ANALYSIS

It is known that for high Prandtl-number fluids heat-transfer takes place within a very thin layer that lies well within the hydrodynamic boundary layer. Authors including Stewartson and Jones [2], Levich [3], Morgan and Warner [4] have shown that the thickness of the temperature layer for free-convection flows is proportional to $\sigma^{-\frac{1}{4}}$. However, the best technique for dealing with such problems fully is provided in [2]. It essentially consists in dividing the whole region into two boundary layers—one of thickness $O(\sigma^{-\frac{1}{4}})$ in which the temperature difference is brought to zero and one of thickness $O(\sigma^{\frac{1}{4}})$ in which the velocity parallel to the surface is brought to zero. For convenience, these two layers will be called the inner and the outer layers respectively. We shall define after [2], different variables as follows.

Inner layer:

$$\left.\begin{aligned}
\zeta_1 &= (3\sigma)^{\frac{1}{4}}\,\eta,\\
f_0 &= (3\sigma)^{-\frac{3}{4}}\,F_0(\zeta_1),\\
\phi_0 &= \Phi_0(\zeta_1),\\
\phi_1 &= \Phi_1(\zeta_1).
\end{aligned}\right\} \quad (3)$$

Outer layer:

$$\left.\begin{aligned}
\zeta_2 &= \gamma(3\sigma)^{-\frac{1}{4}}\,\eta,\\
f_0 &= \gamma(3\sigma)^{-\frac{1}{4}}\,G_0(\zeta_2),\\
\phi_0 &= \phi_1 = 0,
\end{aligned}\right\} \quad (4)$$

where γ is a suitable constant to be specified later. The corresponding equations are

$$F_0''' + \Phi_0 + \frac{1}{3\sigma}\{3F_0F_0'' - 2(F_0')^2\} = 0, \quad (5)$$

$$\Phi_0'' + F_0\Phi_0' = 0, \quad (6)$$

$$\Phi_1'' + F_0\Phi_1' - \tfrac{4}{3}F_0'\Phi_1 + \tfrac{1}{3}(F_0'')^2 = 0, \quad (7)$$

$$G_0''' + 3G_0G_0'' - 2(G_0')^2 = 0. \quad (8)$$

In the above, a prime means differentiation with respect to the appropriate independent variable. Evidently the boundary condition at infinity is redundant for F_0, also those on the surface for G_0. By the method of matched asymptotic solutions it is found that solutions in the following form exist:

$$F_0 = F_{00} + (3\sigma)^{-\frac{1}{4}} F_{01} + \ldots.$$

$$\Phi_0 = \Phi_{00} + (3\sigma)^{-\frac{1}{4}} \Phi_{01} + \ldots.$$

$$\Phi_1 = \Phi_{10} + (3\sigma)^{-\frac{1}{4}} \Phi_{11} + \ldots.$$

$$G_0 = G_{00} + (3\sigma)^{-\frac{1}{4}} G_{01} + \ldots.$$

The relevant boundary conditions are

$F_{00}(0) = 0, \quad F'_{00}(0) = 0, \quad F''_{00}(\infty) = 0;$

$G_{00}(0) = 0, \quad G'_{00}(0) = 1, \quad G'_{00}(\infty) = 0;$

$F_{01}(0) = 0, \quad F'_{01}(0) = 0, \quad F''_{01}(\infty) = \gamma^3 G''_{00}(0);$

$$G_{01}(0) = \frac{1}{\gamma} F_{00}(\infty) - \gamma\zeta_{1\infty},$$

$$G'_{01}(0) = \frac{1}{\gamma^2}\{F'_{01}(\infty) - \zeta_{1\infty} F''_{01}(\infty)\},$$

$G'_{01}(\infty) = 0;$

$\Phi_{00}(0) = 1, \quad \Phi_{00}(\infty) = 0; \quad \Phi_{01}(0) = 0, \quad \Phi_{01}(\infty) = 0;$

$\Phi_{10}(0) = 0, \quad \Phi_{10}(\infty) = 0; \quad \Phi_{11}(0) = 0, \quad \Phi_{11}(\infty) = 0,$

where $\gamma^2 = F'_{00}(\infty)$, $\zeta_{1\infty}$ = the value of ζ_1 at the boundary of the inner layer.

3. SOLUTIONS AND CONCLUSIONS

From numerical computations performed on the electronic computer IBM 7094$^{\text{II}}$ at Imperial College it is found that

$F''_{00}(0) = 1{\cdot}08506, \quad F'_{00}(\infty) = 0{\cdot}88425, \quad \Phi'_{00}(0) = -0{\cdot}54023,$

$F''_{01}(0) = -0{\cdot}70013, \; F'_{01}(\infty) = -5{\cdot}40819, \; \Phi'_{01}(0) = 0{\cdot}24542,$

$G''_{00}(0) = -1{\cdot}54079, \; G''_{01}(0) = -1{\cdot}65434, \; \Phi'_{10}(0) = 0{\cdot}12948,$

$\Phi'_{11}(0) = -0{\cdot}24238.$

The above results show that $\phi'_1(0)/\phi'_0(0)$ attains the value $-0{\cdot}2195$ for $\sigma = 100$ and the value $-0{\cdot}2377$ for $\sigma = 10000$ compared to the values $-0{\cdot}2226$ and $-0{\cdot}2378$ obtained by Gebhart. As $\sigma \to \infty$ this ratio becomes $-0{\cdot}2397$. This is the value sought by Gebhart.

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